IMPROVED MRI RECONSTRUCTION FROM REDUCED SCANS K-SPACE BY INTEGRATING NEURAL PRIORS IN THE BAYESIAN RESTORATION.

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Abstract. **The goal of this paper is to present the development of a new reconstruction methodology for restoring Magnetic Resonance Images (MRI) from reduced scans in k-space. The proposed approach considers the combined use of Neural Network models and Bayesian restoration, in the problem of MRI image extraction from sparsely sampled k-space, following several different sampling schemes, including spiral and radial. Effective solutions to this problem are indispensable especially when dealing with MRI of dynamic phenomena since then, rapid sampling in k-space is required. The goal in such a case is to make measurement time smaller by reducing scanning trajectories as much as possible. In this way, however, underdetermined equations are introduced and poor image reconstruction follows. It is suggested here that significant improvements could be achieved, concerning quality of the extracted image, by judiciously applying Neural Network and Bayesian estimation methods to the k-space data. More specifically, it is demonstrated that Neural Network techniques could construct efficient priors and introduce them in the procedure of Bayesian reconstruction. These ANN Priors are independent of specific image properties and probability distributions. They are based on training supervised Multilayer Perceptron (MLP) neural filters to estimate the missing samples of complex k-space and thus, to improve k-space information capacity. Such a neural filter based prior is integrated to the maximum likelihood procedure involved in the Bayesian reconstruction. It is found that the proposed methodology leads to enhanced image extraction results favorably compared to the ones obtained by the traditional Bayesian MRI reconstruction approach as well as by the pure MLP based reconstruction approach.**

I. INTRODUCTION

A data acquisition process is needed to form the MR images. Such data acquisition occurs in the spatial frequency (**k**-space) domain, where sampling theory determines resolution and field of view, and it results in the formation of the k-space matrix. Strategies for reducing image artifacts are often best developed in this domain. After obtaining such a **k**-space matrix, image reconstruction involves fast multi-dimensional Inverse Fourier transforms, often preceded by data interpolation and re-sampling.

Sampling the **k**-space matrix occurs along suitable trajectories [1]. Ideally, these trajectories are chosen to completely cover the k-space according to the Nyquist sampling criterion. The measurement time of a single trajectory can be made short. However, prior to initiating a trajectory, return to thermal equilibrium of the nuclear spins needs to be awaited. The latter is governed by an often slow natural relaxation process that is beyond control of the scanner and impedes fast scanning. Therefore, the only way to shorten scan time in MRI when needed, as for instance in functional MRI, is to reduce the overall waiting time by using fewer trajectories, which in turn should individually cover more of k-space through added curvatures. Although, however, such trajectory omissions achieve the primary goal, i.e. more rapid measurements, they entail undersampling and violations of the Nyquist criterion thus, leading to concomitant problems for image reconstruction.

The above mentioned rapid scanning in MRI problem is highly related with two other ones. The first is the selection of the optimal scanning scheme in k-space, that is the problem of finding the shape of sampling trajectories that more fully cover the k-space using fewer number of trajectories. Mainly three such alternative shapes have been considered in the literature and are used in actual scanners, namely, Cartesian, radial and spiral [1], associated with different reconstruction techniques. More specifically, the Cartesian scheme uses the inverse 2D FFT, while the radial and spiral scanning involve the Projection Reconstruction, the linogram or the SRS-FT approaches [1].

The second one is associated with image estimation from fewer samples in k-space, that is the problem of omitting as many trajectories as possible without attaining worse reconstruction results. The main result of such scan trajectories omissions is that we have fewer samples in k-space than needed for estimating all pixel intensities in image space. Therefore, there is an infinity of MRI images satisfying the sparse k-space data and thus, the reconstruction problem becomes ill-posed. Additionally, omissions usually cause violation of the Nyquist sampling condition. Despite the fact that solutions are urgently needed, in functional MRI for instance, very few research efforts exist in the literature. The most obvious and simplest such method is the so called "zero-filling the k-space", that is, all missing points in k-space acquire complex values equal to zero. Subsequently, image

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Abstract

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reconstruction is achieved as usually, by applying the inverse Fourier transform to the corresponding k-space matrix. Instead of zero-filling the k-space it might be more advantageous to interpolate it by using nonlinear interpolation procedures, like neural networks, as proposed by the authors [2]. The Bayesian reconstruction approach, developed by two of the authors [1], briefly presented in the next section is another alternative solution. Both these last two mentioned MR reconstruction solutions yield good results [1,2]. The main contribution, however, of this paper is to develop a novel MR reconstruction methodology by involving both Bayesian and Neural reconstruction techniques and present its competence and advantages over the other rival approaches.

II. THE BAYESIAN RECONSTRUCTION APPROACH.

The Bayesian reconstruction approach recently proposed by two of the authors [1], attempts to provide solutions through regularizing the problem by invoking general prior knowledge in the context of Bayesian formalism. The algorithm amounts to minimizing the following objective function [1], by applying the conjugate gradients method,

$$| \underline{\mathbf{S}} - T\, \underline{\mathbf{I}}|^2 / (2\sigma^2) + (3/2) \sum_{x,y} \log \{ \alpha^2 + ({}^{x}\Delta_{xy})^2 + ({}^{y}\Delta_{xy})^2 \} \quad (1)$$

with regards to $\underline{\mathbf{I}}$, which is the unknown image to be reconstructed that fits to the sparse k-space data given in $\underline{\mathbf{S}}$. The first term comes from the likelihood term and the second one from the prior knowledge term of the Bayesian formulation [1]. In the above formula, $T((k_x, k_y),(x,y)) = e^{-2\pi i(xk_x + yk_y)}$ represents the transformation from image to k-space data (through 2-D FFT). The second term symbols arise from the imposed 2D Lorentzian prior knowledge. ${}^{x}\Delta_{xy}$ and ${}^{y}\Delta_{xy}$ are the pixel intensity differences in the x- and y- directions respectively and α is a Lorentz distribution-width parameter. Assuming that P(I) is the prior, imposing prior knowledge conditions for the unknown MRI image, then, the second term of (1) comes as follows.

The starting point is that P(I) could be obviously expanded into $P(I)=P(I_{0,0})\, P(I_{1,0}| I_{0,0})\, P(I_{2,0}| I_{0,0}, I_{1,0})\ldots$ If, now, it is assumed that the intensity $I_{x,y}$ depends only on its left neighbour ($I_{x-1,y}$), then the previous P(I) expansion takes on the form $P(I) = \prod_{(x,y)} P(I_{x,y}| I_{x-1,y})$, provided that the boundaries are ignored. Next, we assume that $P(I_{x,y}| I_{x-1,y})$ is a function only of the difference between the corresponding pixels. This difference is written down as ${}^{x}\Delta_{xy} = I_{x,y} - I_{x-1,y}$. It has been shown that the probability density function of ${}^{x}\Delta_{xy}$ is Lorentzian shaped (see [1]). These assumptions and calculations lead to computing the prior knowledge in the Bayesian reconstruction as in the second term of (1).

Although this Bayesian reconstruction approach tackles the problem of handling missing samples in k-space, it exhibits, however, the disadvantage that assumes the existence of special probability distributions, given in closed form descriptions, for representing the unknown ones occurred in MRI, which is an issue under question. In this paper we attempt to remedy this problem by proposing additional priors in the Bayesian formulation in order to capture the probability distribution functions encountered in MRI. These priors are constructed through applying a specifically designed Multilayer Perceptron (MLP) neural filter for interpolating the sparsely sampled k-space.

III. DESIGN OF MLP NEURAL NETWORK PRIORS

The method herein suggested for designing efficient Priors for the Bayesian reconstruction formalism, is based on the attempt to extract prior knowledge from the process of filling in the missing complex values in k-space from their neighboring complex values. Thus, instead of assuming a Lorentzian prior knowledge to be extracted from the neighboring pixel intensities in MRI, as a constraint to be applied in the conjugate gradient based Bayesian reconstruction process, the proposed strategy doesn't make any assumption. Instead, it aims at extracting priors without any specific consideration concerning the shape of the distributions involved, by transforming the original reconstruction problem into an interpolation one in the complex domain. While linear interpolators have already been used in the literature [2] and nonlinear estimations are well established in MRI [2], ANN models offer several advantages when applied as sparsely sampled k-space interpolators [2]. The methodology to extract prior knowledge by applying the ANN filters in MRI reconstruction is described in the following paragraphs.

Step1. We compile a set of R representative N X N MRI images with k-space matrices completely known, which comprise the training set of the MLP interpolators. Subsequently, we scan these matrices following the specific sampling schemes mentioned above and then, by randomly omitting trajectories the sparse k-spaces are produced.

Step2. The original k-space matrix as well as its corresponding sparse k-space matrix associated with one N X N MRI training image, is raster scanned by a (2M+1) X (2M+1) sliding window containing the associated complex k-space values. The estimation of the complex number in the center of this window from the rest of the complex numbers comprising it is the goal of the proposed interpolation procedure. Each position of this sliding window is, therefore, associated with a desired output pattern comprised of the complex number in the original k-space corresponding to the window position, and an input pattern comprised of the complex numbers in k-space corresponding to the rest (2M+1) X (2M+1) -1 window points.

Step3. Each such pattern is then, normalized according to the following procedure. First, the absolute values of the complex numbers in the input pattern are calculated and then, their average absolute value $|z_{aver}|$ is used to normalize all the complex numbers belonging both in the input and the desired output patterns. That is, if z_i is such a number then this

normalization procedure transforms it into the $z_i/|z_{aver}|$. In the case of test patterns we apply the same procedure. That is, the average absolute value $|z_{aver}|$ for the complex numbers z_i of the test input pattern is first calculated. Then, the normalized complex values $z_i/|z_{aver}|$ feed the MLP interpolation filter to predict the sliding window central normalized complex number z^{norm}_{centre}. The corresponding unnormalized complex number is simply $z^{norm}_{centre} * |z_{aver}|$.

Step4. The next step is the production of training patterns for the MLP interpolators and their training procedure. To this end, by randomly selecting sliding windows from the associated k-spaces of the R training images and producing the corresponding input and desired output training pairs of patterns, as previously defined, we construct the set of training patterns. The assumption underlying such an approach of training MLP interpolators is that there are regularities in every k-space sliding window, the same for any MRI image, to be captured by the MLPs without any prior assumption for the probability distributions. MLP training follows by applying the conjugate gradient technique of Polak-Ribiere.

Step5. After training phase completion, the MLP filter has been designed and can be applied to any similar test MRI image as follows. To this end, the (2M+1) X (2M+1) sliding window raster scans the sparse k-space matrix associated with this test image, starting from the center. Its central point position moves along the perimeter of rectangles covering completely the k-space, having as center of gravity the center of the k-space array and having distance from their two adjacent ones of 1 pixel. It can move clockwise or counterclockwise or in both directions. For every position of the sliding window, the corresponding input pattern of (2M+1) X (2M+1) – 1 complex numbers is derived following the above described normalization procedure. Subsequently, this normalized pattern feeds the MLP interpolator. The wanted complex number corresponding to the sliding window center, is found as $z_{centre} = z_{MLP}^{out} * |z_{aver}|$, where z_{MLP}^{out} is the MLP output and $|z_{aver}|$ the average absolute value of the complex numbers comprising the unnormalized input pattern. For each rectangle covering the k-space, the previously defined filling in process takes place so that it completely covers its perimeter, only once, in both clockwise and counterclockwise directions. The final missing complex values are estimated as the average of their clockwise and counter-clockwise obtained counterparts. The outcome of the MLP filter application is the reconstructed test image, herein named MLP_Img (equation (2)). Its difference from the image $I^{(t)}$ obtained during the previous step of conjugate gradient optimization in the Bayesian reconstruction formula (1), provides the neural prior to be added for the current optimization step.

IV. INCORPORATION OF NEURAL NETWORK PRIOR KNOWLEDGE INTO THE BAYESIAN PRIOR

Following the 5 steps above we can formulate the incorporation of MLP priors to the Bayesian restoration process as follows.

• Design the MLP Neural filter as previously defined

• Consider the Bayesian reconstruction formula (1). The image to be optimized is I given the k-space S. The initial image in the process of conjugate gradient optimization is the zero-filled image. At each step t of the process a different I(t) (the image at the t step, that is, the design variables of the problem) is the result. Based on fig. 1, by applying the MLP filter on the original sparse k-space, but with the missing points initially filled by the FFT of I(t) (in order to derive the I(t) k-space)- and afterwards refined by the MLP predictions, we could obtain the difference I(t) -MLP_Img(t) as the Neural Prior.

• The Neural Network (NN) Prior form, therefore, is:

$$\sum_{x,y} | MLP\_\operatorname{Im} g^{(t)}(x,y) - I^{(t)}(x,y) | \qquad (2)$$

where, $MLP\_Img^{(t)}(x,y)$ is the NN estimated pixel intensity in image space (NN reconstructed image: Inverse FFT of NN completed k-space) at step t and $I^{(t)}(x,y)$ is the image obtained at step t of the conjugate gradient optimization process in the Bayesian reconstruction

• The proposed Prior in the Bayesian reconstruction is given as

Final Prior = Lorentzian Bayesian Prior + a* NN_Prior (3)

• That is, the optimization process $I^{(t)}$ is attempted to be guided by the $MLP\_Img^{(t)}$ produced by the NN

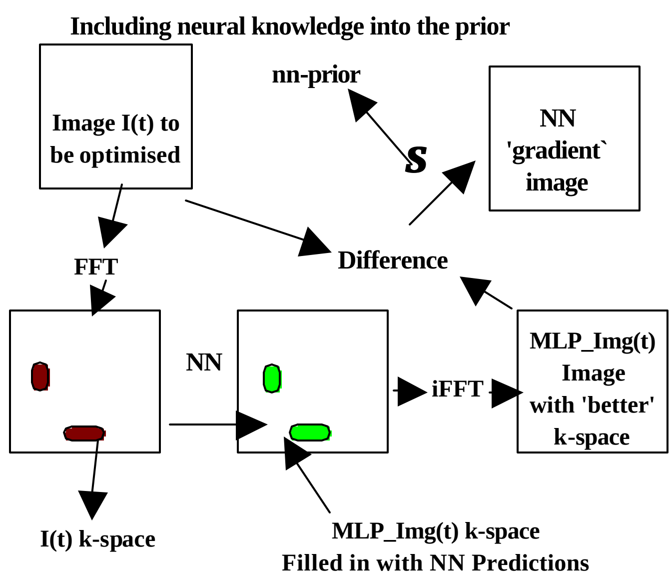

Figure 1. The difference between the image to be optimized I(t) and the MLP reconstructed image MLP_Img(t) constitutes the neural prior.

V. EVALUATION STUDY AND CONCLUSIONS

An extensive experimental study has been conducted in order to evaluate the above defined novel Bayesian reconstruction methodology. All the methods involved have been applied to a large MRI image database downloaded from the Internet, namely, the Whole Brain Atlas http://www.med.harvard.edu/AANLIB/home.html (copyright © 1995-1999 Keith A. Johnson and J. Alex Becker). We have used 10 images randomly selected out of this collection for training the MLP filters, and 10 images, again randomly selected for testing the proposed and the rival reconstruction methodologies. All images are 256 by 256. Their k-space matrices have been produced applying the 2D FFT to them. Radial trajectories have been used to scan the resulted 256 X 256 complex k-space arrays. 4 X 256 = 1024 radial trajectories are needed to completely cover such k-spaces. In order to apply the reconstruction techniques involved in this study, each k-space has been sparsely sampled using 128 only radial trajectories. Regarding the sliding window raster scanning the k-space, a 5 X 5 window was the best selection.

Concerning the MLP filter architecture, the 48-10-2 one was found to be the best. This MLP has been trained using 3600 training patterns. The compared reconstruction techniques involved in this study are: the proposed novel Bayesian reconstruction approach, the traditional Bayesian reconstruction technique as well as the MLP filtering interpolation technique. Moreover, the simplest "interpolation" approach, namely filling in the missing samples in k-space with zeroes and then, reconstructing the image, has been invoked. All these methods have been implemented using the MATLAB programming platform.

Concerning the measures involved to quantitatively compare reconstruction performance, we have employed the usually used Sum of Squared Errors (SSE) between the original MRI image pixel intensities and the corresponding pixel intensities of the reconstructed image. Additionally, another quantitative measure has been used, which expresses performance differences in terms of the RMS error in dB [2]:

```
lambda=(image_recon(:)'*image_orig(:))/(image_recon(:)'*image_recon(:));residu=image_orig-lambda*image_recon;
dB=10*log10((image_orig(:)'*image_orig(:))/(residu(:)'*residu(:)));
```

The quantitative results obtained by the different reconstruction methods involved are outlined in table 1 (average SSE and RMS errors for the 10 test MRI images). Concerning reconstruction performance qualitative results, a sample is shown in figure 2. Both quantitative and qualitative results clearly demonstrate the superiority of the proposed Bayesian reconstruction methodology embedding MLP filtering based prior knowledge, in terms of MRI image reconstruction performance over the other three rival methodologies (simple Bayesian, MLP filter and zero-filled reconstructions). Future trends of our research effort include implementation of the 3-D Bayesian reconstruction with NN-priors for f-MRI as well as applications in MRI image segmentation for tumor detection.

Method	**SSE** (average in the 10 test MRI images)	**dB** (average in the 10 test MRI images)
Proposed Bayesian reconstruction with NN Prior	2.85 E3	16.67
Bayesian reconstruction	3.40e3	15.92
MLP filtering	3.30E3	15.98
Zero-filling reconstruct.	3.71E3	15.26

Table 1. The quantitative results with regards to reconstruction performance of the various methodologies involved

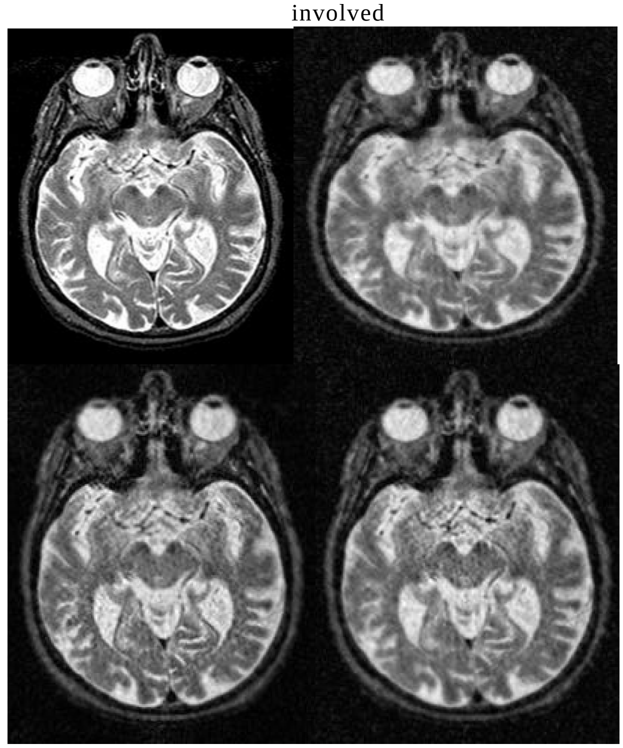

Fig. 2 From left to right and top to down: The proposed Bayesian with NN prior, the sparsely sampled k-space (nr=128)–zerofilled image reconstruction, the MLP filtering and the traditional Bayesian reconstruction results. The Test Image illustrates a brain slice with Alzheimer's disease (http://www.med.harvard.edu/ AANLIB/cases/case3/mr1-tc1/020.html).

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